THE ART OF COMBINATORICS - VOLUME III

ORDER AND OPTIMIZATION

©Douglas B. West

Introduction 2
- Orders and graphs. 2
- Subposets. 5
- Examples and operations. 8
- Exercises. 11

Chapter 11 - Structure of Posets 13

11.1 - Saturated partitions. 14
- Dilworth's Theorem. 14
- The Greene-Kleitman Theorem. 19
- Further topics. 27
- Exercises. 30

11.2 - Sperner's Theorem and LYM orders. 34
- The LYM property. 35
- Products of LYM orders. 41
- Sperner properties. 43
- Generalizations. 45
- Exercises. 46

11.3 - Symmetric chain orders. 50
- Sets and multisets. 51
- Applications. 55
- Other symmetric chain orders. 60
- Unimodality (under construction). 68
- Exercises. 71

11.4 - Lattices. 74
- Examples and basic properties. 74
- Distributive lattices. 79
- Modular lattices. 84
- Exercises. 90

11.5 - Posets and graphs. 94
- Comparability graphs. 94
- Canonical decomposition. 100
- Cover graphs and diagrams. 105
- Exercises. 107

Chapter 12 - Linear Extensions 111

12.1 - Preference and Measurement. 112
- Rankings and consensus. 112
- Semiorders and interval orders. 117
- Ferrers relations (biorders). 122
- Exercises. 123

12.2 - Order dimension. 126
- Definitions and examples. 126
- Characterization and computation. 130
- Bounds and removal theorems. 134
- Interval and Ferrers dimension (optional). 140
- Restricted realizers (optional). 145
- Exercises. 147

12.3 - Dimension of special classes. 154
- Bipartite posets. 154
- Dimension and planarity. 163
- Containment orders. 166
- Exercises. 172

12.4 - Correlational inequalities. 174
- FKG and Ahlswede-Daykin inequalities. 174
- The XYZ Inequality. 179
- Height distribution. 185
- Exercises. 186

12.5 - Sorting and searching. 190
- Balanced comparisons. 190
- Explicit algorithms. 200
- Order statistics. 202
- Searching in posets. 204
- Exercises 210

Chapter 13 - Extremal & Enumerative Problems

13.1 - Families of subsets. 214
- Intersecting families. 214
- The Erdős-Ko-Rado Theorem. 216
- The Kruskal-Katona Theorem. 220
- Applications of linear algebra. 225
- Order ideals & Chvátal's Conjecture. 227
- Sunflowers. 231
- Exercises. 232

13.2 - Recursive ordered sets. 236
Recursive models. 236
Antichain partitions. 238
Chain covers. 242
Exercises. 252

13.3 - Miscellaneous extremal problems. 254
Linear discrepancy. 254
Weak and fractional weak discrepancy. 266
Cutsets and fibres. 268
Regressions and monotone chains. 274
Simple games and winning coalitions (mostly unwritten). 276
Exercises. 277

13.4 - Möbius inversion. 278
The incidence algebra. 280
The Möbius function and inversion. 283
A taste of reciprocity (optional). 289
Binomial posets (optional). 295
Structure of posets (under construction). 300
Exercises. 302

Chapter 14 - Linear & Integer Programming 305

14.1 - Linear programming. 306
Packing and covering. 306
Linear programs and duality. 309
The Simplex Algorithm. 312
Farkas' Lemma and Second Neighborhoods. 319
Shannon capacity and strong products. 323
Matrix games. 327
Combinatorial applications. 332
Exercises. 337

14.2 - Network flow. 340
The standard network model. 340
Combinatorial applications. 347
Supplies and demands. 353
Minimum cost flow. 357
Exercises. 366

14.3 - Matrices and polyhedra. 372
Clutters and blockers. 372
T-cuts and T-joins. 378
Totally unimodular matrices. 384
Total dual integrality. 391
Cutting planes. 398

Branch-and-bound (unwritten). 402
Exercises. 402

Chapter 15 - Matroids 401

15.1 - Examples and properties. 406
Hereditary systems and examples. 406
Axiomatics of matroids. 413
The span function. 418
Duality and minors. 422
Exercises. 429

15.2 - Matroids and optimization. 436
Matroid intersection. 436
Matroid union. 441
Applications of matroid union. 444
Matroid intersection algorithm. 450
Matroid parity and matroid matching. 455
Polymatroid rank functions. 457
Polymatroids. 461
Polymatroidal network flow. 464
Exercises. 472

15.3 - Matroids and graphs. 476
Connected matroids. 476
Whitney's 2-Isomorphism Theorem. 479
Gammoids, linkages, and Menger's Theorem 485
Binary and linear matroids. 490
Matroid basis graph. 497
Exercises. 499

15.4 - From lattices to greedoids. 502
Matroids and lattices. 502
Convexity and antimatroids. 509
Circuits in antimatroids. 515
Languages and greedoids. 522
Rank and span in greedoids. 526
Interval greedoids. 529
Diameter of greedoids. 532
Exercises. 542

15.5 - Oriented Matroids (mostly unwritten). 546
Exercises. 551

Appendix 1 - Glossary of Terms 552
Appendix 2 - Summary of Notation 584
Appendix 3 - Index of Definitions 590