Mathematics Department - University of Illinois

MATH 312 - SYLLABUS FOR INSTRUCTORS

Text - West, *Introduction to Graph Theory,* Prentice Hall, Second Edition (2000).

Many students in this course see graph algorithms repeatedly in courses in computer science. Hence this course aims primarily to improve students' writing of proofs in discrete mathematics while learning about the structure of graphs. Some algorithms are presented along the way, and many of the proofs are constructive. The aspect of algorithms emphasized in CS courses is running time; the focus in a mathematics course in graph theory from this book is on proving that the algorithms work.

Math 312 is intended as a rigorous course that challenges students to think. Homework and tests should require proofs, and most of the exercises in the text do so. The material is interesting, accessible, and applicable; most students who stick with the course will give it a fair amount of time and thought. Solutions for exercises are available from the author, and there is a solution manual available from George Lobell (`george_lobell@prenhall.com`) at Prentice Hall.

Suggested Schedule

The subject matter for the course is the first seven chapters of the text, skipping most optional material. Modifications to this are discussed below. Slightly under two lectures on average are allotted to each of the 22 sections.

In the exercises, problems designated by $(-)$ are easier or shorter than most, often good for tests or for "warmup" before doing homework problems. Problems designated by $(+)$ are harder than most. Those designated by (!) are particularly instructive, entertaining, or important.

Chapter 1	Fundamental Concepts	8
Chapter 2	Trees and Distance	5
Chapter 3	Matchings and Factors	5
Chapter 4	Connectivity and Paths	6
Chapter 5	Graph Coloring	6
Chapter 6	Planar Graphs	5
Chapter 7	Edges and Cycles	5
*	Exams, Leeway, or Optional	3
*	Total	43

Optional material to include

No later material requires material marked optional. The "optional" marking also indicates to students that the final examination is unlikely to test on this material.

The optional subsections on Disjoint Spanning Trees (Bridg-It) in Section 2.1 and Stable Matchings in Section 3.2 are always quite popular with the students. The planarity algorithm (without proof) in 6.2 is appealing to students, as is the notion of embedding graphs on the torus through Example 6.3.21. Our course usually includes these four items.

The reduction of f-factors to 1-factors in Section 3.3 is also very instructive and is covered when the class is proceeding on schedule. Other potential additions include the applications of Menger's Theorem at 4.2.24 or 4.2.25.

In general, other items marked optional should not be presented in class.

Additional text items not marked optional that can be skipped when behind schedule:
1.1: 31, 35. 1.2: 16, 21-23. 1.3: 24, 31-32. 1.4: 1, 4, 7, 25-26.
2.1: 8, 14-16. 2.2: 13-19. 2.3: 7-8. 3.2: 4. 4.1: 4-6. 4.2: 20-21.
5.1: 11, 22(proof). 5.3: 10-11, 16(proof). 6.1: 18-20, 28. 6.3: 9-10, 13-15. 7.2: 17.

Comments

There are several underlying themes in the course, and mentioning these from time to time helps establish continuity. 1) TONCAS (The Obvious Necessary Condition(s) are Also Sufficient. 2) Weak duality in dual maximation and minimization problems. 3) Proof techniques, especially the use of extremality, the paradigm for inductive proofs of conditional statements, and the technique of transforming a problem into a previously solved problem.

Chapter 1. Undergraduates seem to appreciate the review of proof techniques in the first five sections; graduate students don't need it, so the emphasis on this can be adjusted to the mix of the class. To minimize confusion, digraphs should not be mentioned until section 1.4; students absorb the additional model more easily after becoming comfortable with the first.

1.1: p3-6 contain motivational examples that can be presented on the first day as an overview of the course; this should not extend past the first day (the definitions are later repeated where needed). The material on the Petersen graph establishes its basic properties for use in later examples and exercises.

1.2: The definitions of path and cycle are intended to be intuitive; one shouldn't dwell on the heaviness of the notation for walks.

1.3: Although characterization of graphic sequences is a classical topic, some reviewers have questioned its importance. Nevertheless, here is a computation that students can perform.

1.4: The examples are presented to motivate the model; these can be skipped if needed to save time. The de Bruijn graph is classical motivation for Eulerian digraphs. It should be covered if possible, but it takes a while to explain.

Chapter 2.

2.1: The equivalence of characterizations of trees is a good place to ask for input from the class, both in listing properties and it proving equivalence.

2.2: The inductive proof for the Prüfer correspondence seems to be easier for most students to grasp than the full bijective proof; it also illustrates the usual type of induction with trees. Most students in the class have seen determinants, but most have considerable difficulty understanding the proof of the Matrix Tree Theorem; given the time involved, it is best just to state the result and give an example. Students find the material on graceful labelings enjoyable and illuminating; it doesn't take long, but also it isn't imperative. The material on branchings should certaily be skipped in this course.

2.3: Many students have seen rooted trees in computer science and find ordinary trees unnatural; we hope that Kruskal's algorithm will clarify the distinction. Many CS courses now cover the algorithms of Kruskal, Dijkstra, and Huffman; cover Kruskal, maybe cover Dijkstra (ask whether students have seen it), and skip Huffman.

Chapter 3.

3.1: Skip "Dominating Sets", but present the rest of the section.

3.2: Students find the Hungarian algorithm difficult; explicit examples need to be worked along with the theoretical discussion of the equality subgraph. "Stable Matchings" is very popular with students and should be presented unless far behind in schedule. Skip "Faster Bipartite Matching".

3.3: Present all of the subsection on Tutte's 1-factor Theorem. The material on f-factors is intellectually beautiful and leads to one proof of the Erdős-Gallai conditions, but it is not used again in the course and is an "aside". Skip everything on Edmonds' Blossom Algorithm: matching algorithms in general graphs are important algorithmically but would require too much time in this course; this is "recommended reading".

Chapter 4.

4.1: Students have trouble distinguishing "k-connected" vs. "connectivity k" and "bonds" vs. "edge cuts". Bonds are dual to cycles in the matroidal sense; there are hints of this in exercises and in Chapter 7, but it cannot really be explored until Chapter 8.

4.2: Students find this section a bit difficult. The proof of 4.2.10 is similar to that of 4.2.7, making it omittable, but the application in 4.2.14 is appealing. The details of 4.2.20-21 can be skipped or treated lightly, since the main issue is the Menger theorem for the local duality. 4.2.24-25 are appealing applications that can be added; 4.2.5 (CSDR) is a fundamental result but takes a fair amount of effort.

4.3: The material on network flow is quite easy but can take a long time to present due to the overhead of defining new concepts. The basic idea of 4.3.13-15 should be presented without belaboring the details too much. 4.3.16 is a more appealing application that perhaps makes the point more effectively. Skip "Supplies and Demands".

Chapter 5.

5.1: If time is short, the proof of 5.1.22 (Brooks' Theorem) can be merely sketched.

5.2: Presentation of Dirac's 5.2.20 is valuable as an application of the Fan Lemma (Menger's Theorem). The subsequent material has limited appeal to undergraduates.

5.3: The inclusion-exclusion formula for the chromatic polynomial is derived here (5.3.10) without using inclusion-exclusion, making it accessible to this class without prerequisite. Nevertheless, this is a good place to save time if needed. Chordal graphs and perfect graphs are important topics in modern applied graph theory, but this is a possible place to tread lightly if short of time. Skip "Counting Acyclic Orientations".

Chapter 6.

6.1: The technical definitions of objects in the plane should be treated very lightly. It is better to be informal here, without writing out formal definitions unless explicitly requested by students. Outerplanar graphs are useful as a much easier class in which to solve problems like those on planar graphs; 6.18-20 are fundamental observations about these graphs, but other items are more important if time is short. 6.1.28 (polyhedra) is an appealing application but can be skipped.

6.2: The preparatory material 6.2.4-7 on Kuratowski's Theorem can be presented lightly, leaving the annoying details as reading; the subsequent material on convex embedding of 3-connected graphs is much more interesting. Presentation of the planarity algorithm is optional; don't prove that it works.

6.3: The four color problem is popular; for undergraduates, show the flaw in Kempe's proof (p271), but don't present the discharging rule unless ahead of schedule. Students find the problem of crossing number easy to grasp, but the results are fairly difficult; try to go as far as the recursive quartic lower bound for the complete graph. The edge bound and its geometric application are impressive but take too much time for undergraduates. The idea of embeddings on surfaces can be conveyed through the examples in 6.3.21 on the torus. Interested students can be advised to read the rest of this section.

Chapter 7.

7.1: The proof of Vizing's Theorem is one of the more difficult in the course, but undergraduates can gain some appreciation of it when it is presented with sufficient colored chalk. The proof can also be skipped. Skip "Characterization of Line Graphs", although if time and interest is plentiful the necessary of Krausz's condition can be explained.

7.2: Chvátal's theorem (7.2.13) is not as hard to present as it looks if the instructor has the statement and proof clearly in mind. Nevertheless, it is somewhat technical and can be skipped (the same can be said of 7.2.17). More appealing is the Chvátal–Erdős Theorem (7.2.19), which certainly should be presented. Skip "Cycles in Directed Graphs".

7.3: The theorems of Tait and Grinberg make a nice culmination to the required material of the course. Skip "Snarks" and "Flows and Cycle Covers". Nevertheless, these are lively topics that can be recommended for advanced students.

Chapter 8. If time permits, material from the first part of any section of Chapter 8 can be presented to give the students a glimpse of other topics. This can be done in optional lectures at the end of the course if evening exams have been given. The best choices for conveying some understanding in a brief treatment are Section 8.3 (Ramsey Theory or Sperner's Lemma) and Section 8.5 (Random Graphs). Also possible are the Gossip Problem (or other items) from Section 8.4 and some of the optional material from earlier chapters, especially snarks and flows. Sections 8.1, 8.2, 8.6 require too much investment in preliminary material to be effective for this purpose.