THE ART OF COMBINATORICS

VOLUME I - EXTREMAL GRAPH THEORY

VOLUME II - STRUCTURE OF GRAPHS

VOLUME III - ORDER AND OPTIMIZATION

VOLUME IV - ARRANGEMENTS AND METHODS

THE ART OF COMBINATORICS

SERIES PREFACE

The Art of Combinatorics (TAC) spans a large portion of modern combinatorics. It consists of four texts at the advanced graduate/professional level, plus two volumes of problems and solutions. Written for mathematicians and for theoretical computer scientists who use discrete mathematics, it provides textbooks for four advanced graduate courses and a research-level reference with proofs of classical and recent results.

Evoking Donald E. Knuth's *The Art of Computer Programming*, Peter Winkler suggested the title I have used for this series. TAC treats material in more depth than my one-semester introductory texts, *Introduction to Graph Theory* (Prentice Hall, 1996 and 2001) and *Introduction to Combinatorics* (planned), and my two-semester graduate-level introduction *Combinatorial Mathematics* (in preparation). The range of material is similar to that of the *Handbook of Combinatorics* edited by Graham, Lovász, and Schrijver (M.I.T. Press, 1995).

One Subject or Many?

Combinatorics has grown enormously in the past 50 years. Specialists in graph theory, partially ordered sets (posets), matroids, enumeration, designs, etc., may have little contact with each other despite common themes in their work (combinatorial designs provide good constructions for extremal graph problems, containment posets correspond to intersection graphs, matroids and polyhedral combinatorics yield combinatorial min-max relations for graphs and partial orders, etc.). I hope that TAC will encourage reunification, providing a common resource for information in many areas of combinatorics.

Combinatorics has many excellent monographs and specialized references, but I think nothing quite like TAC. Handbooks in particular areas list many known results, and the *Handbook of Combinatorics* is remarkably broad, but TAC includes more detailed proofs due to its textbook format. Many of these proofs have not appeared previously in textbooks.

Research in discrete mathematics has been fueled by computational applications of partially ordered sets, graphs, and combinatorial methods. Theoretical computer scientists use combinatorial structures in designing algorithms, and many discrete mathematicians study applications to computer science. TAC discusses various such applications. These include Yao's application of Ramsey theory to prove the optimality of sorted tables, interactions between sorting problems and partially ordered sets, algorithmic implications of results on matroids and polyhedral combinatorics, and the Robertson-Seymour theory of graph minors and its implications in complexity theory. Probabilistic algorithms and probabilistic analysis of deterministic algorithms have stimulated research on random graphs and on the probabilistic method. The Borsuk-Ulam Theorem of topology has been applied to problems in VLSI theory and computational geometry.

Attention to applications in computer science leads to more emphasis on extremal results (especially for graphs and partial orders) than in classical combinatorics, which would begin with the first chapter of Volume IV. TAC has two volumes on graph theory, one on posets and optimization, and one on methods. This reflects both my own tastes and the relative volume of publication in recent years.

Graduate Textbooks

Each volume of TAC provides a self-contained course, and the four volumes are independent. As textbooks, the volumes of TAC are intended for students who expect to devote a lot of time to combinatorics. The books support the detailed study characteristic of topics courses, but they are broader than typical topics courses, permitting much flexibility in the selection of material. Roughly speaking, in each section the more fundamental material appears earlier; this facilitates design of a broader course. Combinatorics is perhaps unique among mathematical disciplines in the accessibility of its problems and results. This permits a good student to move from a solid undergraduate introduction directly to topics courses.

Nevertheless, because there are many interactions among the various topics in combinatorics, it is helpful for students to start with a broad core course in combinatorics. This permits the advanced courses to rotate independently and helps students decide whether they want to study combinatorics in detail. Also, students in other specialties may seek a one-semester introduction to combinatorics, especially if they have not studied it as undergraduates. At the University of Illinois, we offer such a one-semester course at the graduate level.

Combinatorial Mathematics (CM) is intended to serve this need and to serve as a two-semester core course for schools that don't have regular advanced graduate courses in combinatorics. Initially extracted from the four volumes of TAC, CM overlaps significantly with these volumes but has been tightened and reorganized. It emphasizes the essential results, interactions among various parts of combinatorics, and glimpses of more advanced material.

TAC tackles the more difficult material in earnest. Each volume includes much of the corresponding basic material from CM, treated more concisely and efficiently or in greater generality. When a volume of TAC is used after an introductory course, basic material can be reviewed quickly as needed, with most time spent on details of the advanced material. Thus the four courses we teach from TAC nominally require the one-semester core course or some other appropriate prior exposure to the area treated by the advanced course.

Organization

Volumes I and II concern extremal and structural graph theory, respectively. I long resisted splitting graph theory into two volumes. Gradually the material sorted itself into two semesters according to whether its primary motivation was structural or extremal. The resulting separation works surprisingly well, especially when students have a prior introduction to graph theory. As a one-semester treatment of graph theory I wrote the first seven chapters of *Introduction to Graph Theory*. (Its lengthy final chapter samples from the topics in TAC.) The development of IGT allowed TAC to treat graph theory in two volumes and thus become more thorough.

In Volume III, the underlying fundamental object is the collection of subsets of a set, ordered by inclusion. Generalized, this becomes the topic of partial orders, studied in Chapters 11 and 12. Families of sets lead to extremal problems in Chapter 13, to hypergraphs and integer programming in Chapter 14, and to matroids in Chapter 15. The fundamental min-max relations concerning partial orders in Chapter 11 return in Chapters 14 and 15 with proofs by methods of optimization.

Volume IV collects topics that involve classical combinatorics and interactions between combinatorics and other areas of mathematics. Its focus is broader than that of the other volumes, and thus even moreso it aims to survey methods rather than results. The probabilistic method can be viewed as an extension of counting arguments, and it has significant applications in the areas of Ramsey theory, design theory, and combinatorial geometry. The three latter topics are relatively independent.

The exercises in TAC permit attention to additional results without additional length. This follows the tradition of Bollobás's *Extremal Graph Theory* and Knuth's *The Art of Computer Programming*. Results that are less central and not too lengthy appear as exercises. The exercises come from research papers, survey papers, monographs, earlier texts, etc. By testing the exercises in courses (about 50 problems per semester), I have tried to break them into tractable steps, with hints for more difficult problems. Easier, harder, and especially interesting problems are designated by "(−)", "(+)", "(!)", respectively.

Each volume will have thorough glossaries of notation and terminology and an extensive bibliography and index. Volumes A and B are growing gradually from solution sets to the exercises used in classes. The main results of the corresponding section of the text will appear without proof (for reference as needed), followed by the exercises with hints. The second portion will be solutions. These volumes can be used much like Lovász's *Combinatorial Problems and Exercises*, Tomescu and Melter's *Problems in Combinatorics and Graph Theory*, or Melnikov et al's *Exercises in Graph Theory*. They permit self-study, but with the inclusion of a thorough index they also provide a valuable reference for professionals, collecting proofs of many results that might otherwise be hard to find.

A website will be maintained for corrections and updates.

Pedagogy

As a textbook, TAC presents full proofs for most results. This requires efficient use of space, but many students found early versions terse, so I am gradually adding examples and figures to assist the reader. Notation is used to clarify discussion, not to achieve compactness at the expense of understanding.

The backgrounds of students entering graduate courses in combinatorics vary widely. On the other hand, in combinatorics it is a short distance from basic concepts to sophisticated proofs. It is thus both desirable and possible to make the volumes of TAC self-contained, and this leads to overlap with my lower-level books. Nevertheless, it is expected that readers of TAC have some familiarity with basic combinatorics, so elementary material will not be presented with the detail it would have in an elementary text or in CM, which do not assume prior exposure to combinatorics.

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